Formulating and solving non-standard model types using gams/emp

Jan-Hendrik Jagla jhjagla@gams.com

Michael Ferris ferris@cs.wisc.edu

Alex Meeraus ameeraus@gams.com

GAMS Software GmbH

www.gams.de

GAMS Development Corp.

www.gams.com

EURO 2009 Bonn

Agenda

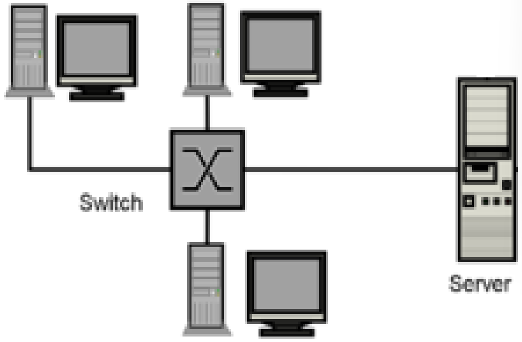
- **G**eneral **A**lgebraic **M**odeling **S**ystem
- New Solution Concepts
- **E**xtended **M**athematical **P**rogramming

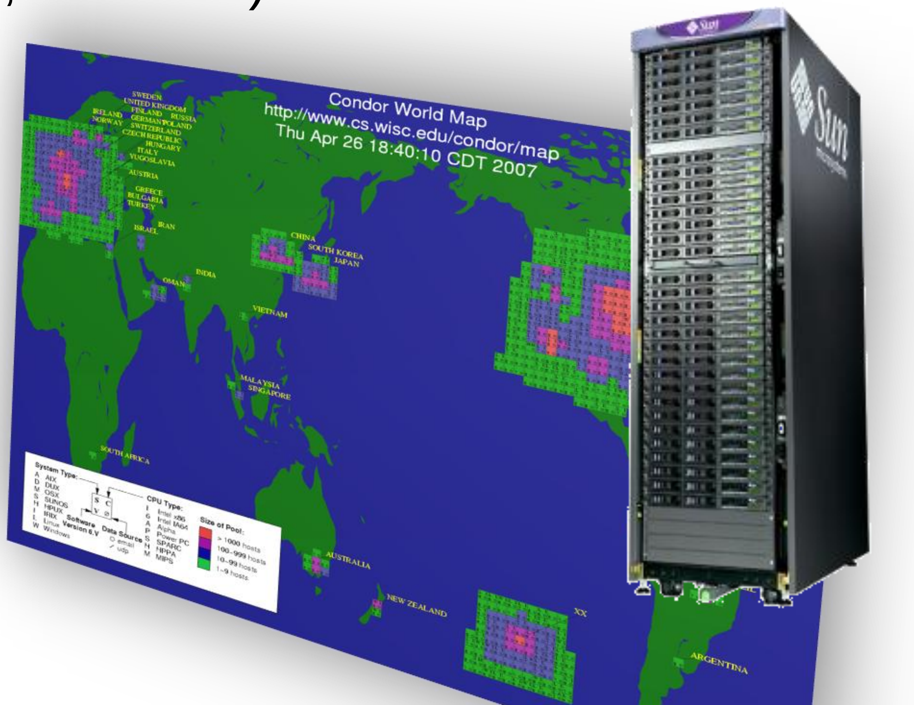
Session ??

Parallel Nonlinear Programming Algorithms

- GAMS supports solvers which make use of multiple threads and/or concurrent strategies
  - **MIP** (CPLEX, GUROBI, XA, XPRESS)
  - **NLP** (CONOPT4, MOSEK)

- GAMS Grid Computing





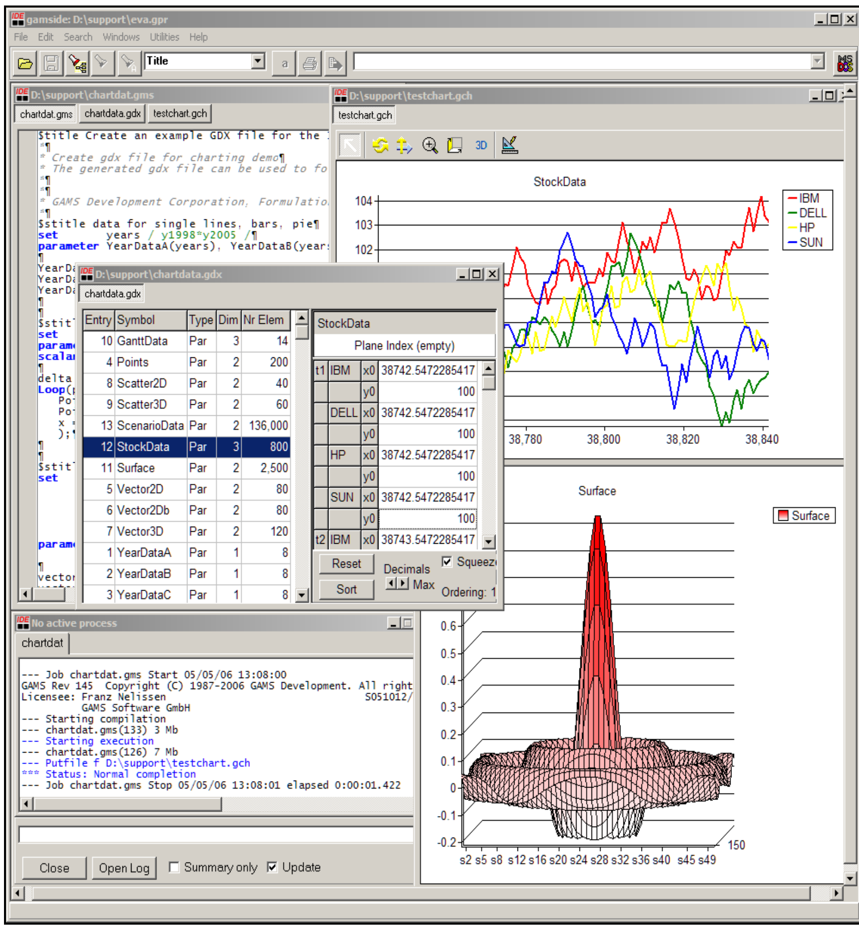
Agenda

General **A**lgebraic **M**odeling **S**ystem

New Solution Concepts

Extended **M**athematical **P**rogramming

GAMS at a Glance



Algebraic Modeling System

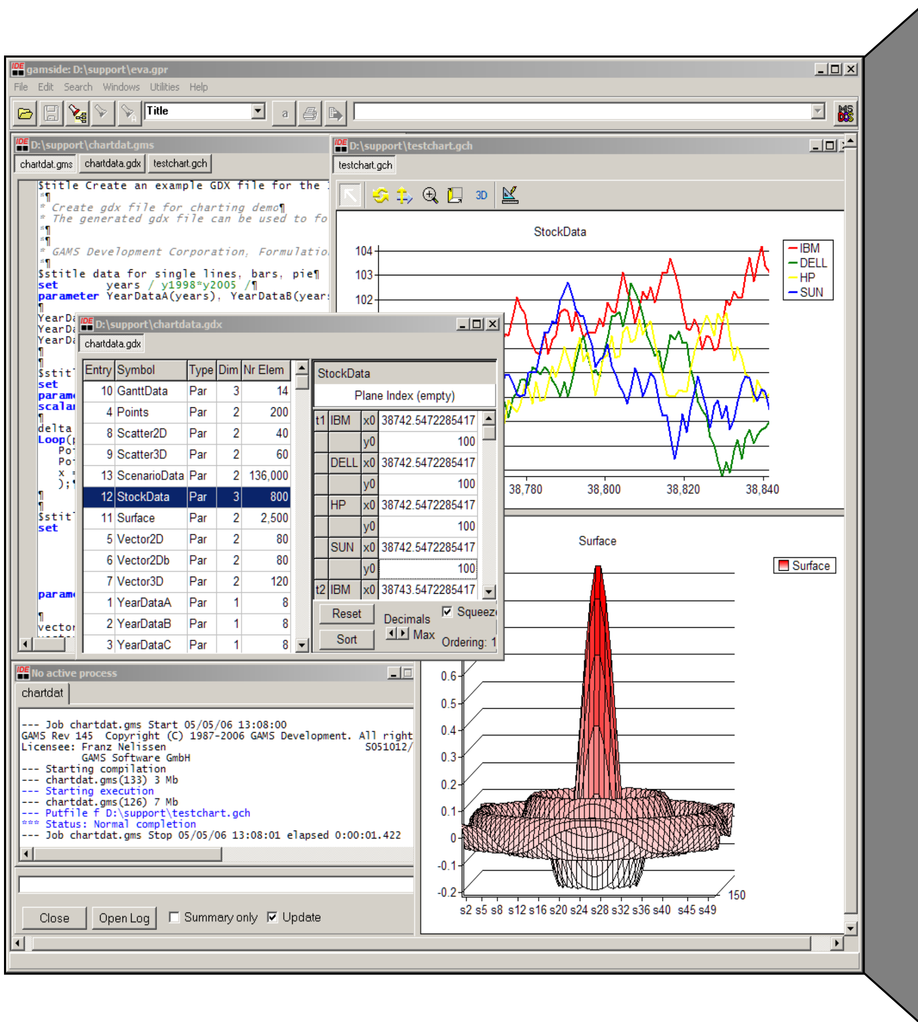
- Facilitates to formulate mathematical optimization problems similar to algebraic notation

  ➔ Simplified model building

- Provides links to appropriate state-of-the-art external algorithms

  ➔ Efficient solution process

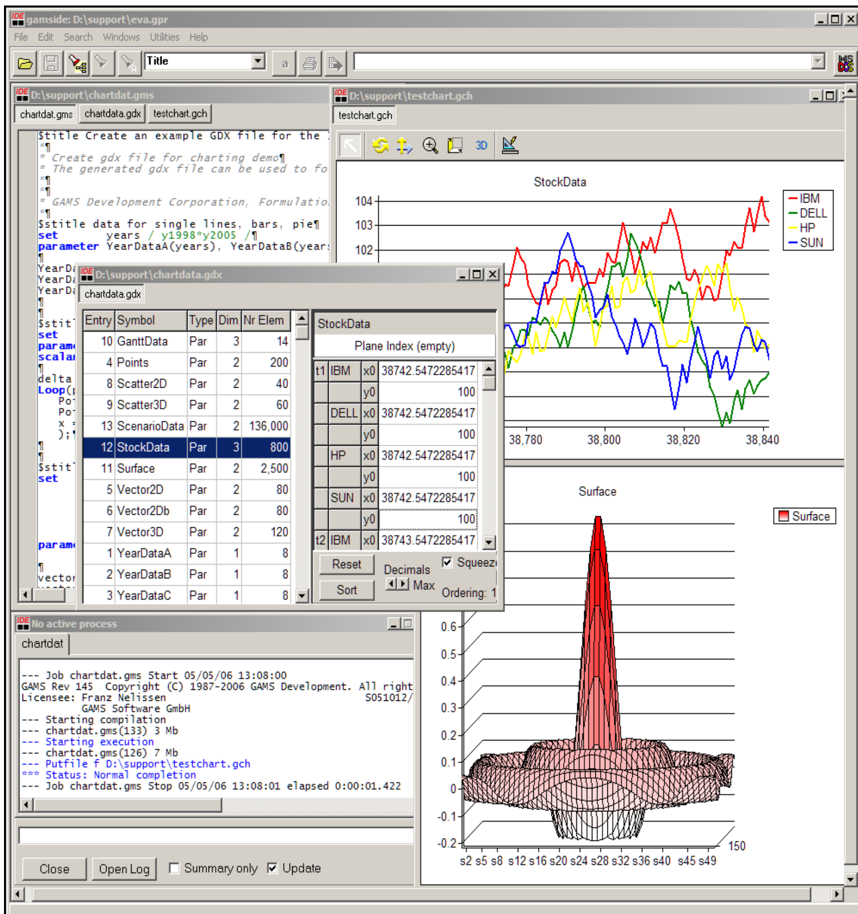
GAMS at a Glance



General **A**lgebraic **M**odeling **S**ystem

- Roots: World Bank, 1976
- Went commercial in 1987
- GAMS Development Corp.
- GAMS Software GmbH
- Broad academic & commercial user community and network

GAMS at a Glance



General **A**lgebraic **M**odeling **S**ystem

- Algebraic Modeling Language
- 25+ Integrated Solvers
- 10+ Supported MP classes
- 10+ Supported Platforms
- Connectivity- & Productivity Tools
  - IDE
  - Model Libraries
  - GDX, Interfaces & Tools
  - Grid Computing
  - Benchmarking
  - Compression & Encryption
  - Deployment System
  - …

Agenda

General **A**lgebraic **M**odeling **S**ystem

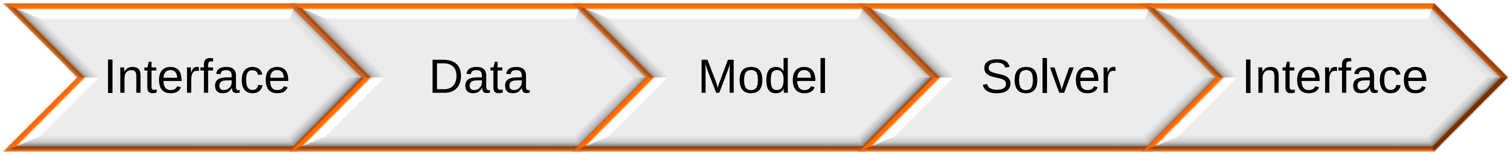
New Solution Concepts

Extended **M**athematical **P**rogramming

Traditional but fundamental concept of AMLs

Different layers with separation of

- model and data
- model and solution methods
- model and operating system
- model and interface



Current state: Model-Side

- Traditional problem format

$$\min_{x} c(x) \quad s.t.\ A_1(x) \le b_1,\ A_2(x) = b_2$$

- Support for complementarity constraints

- Interactions between models possible
  - Series of models
  - Scenario analyses / parallelized model runs
  - Iterative sequential feedback
  - Decomposition

Current state: Solver-Side

Support of a wide collection of established MP classes through solver cluster!

➔ **Tremendous algorithmic and computational progress**

LP

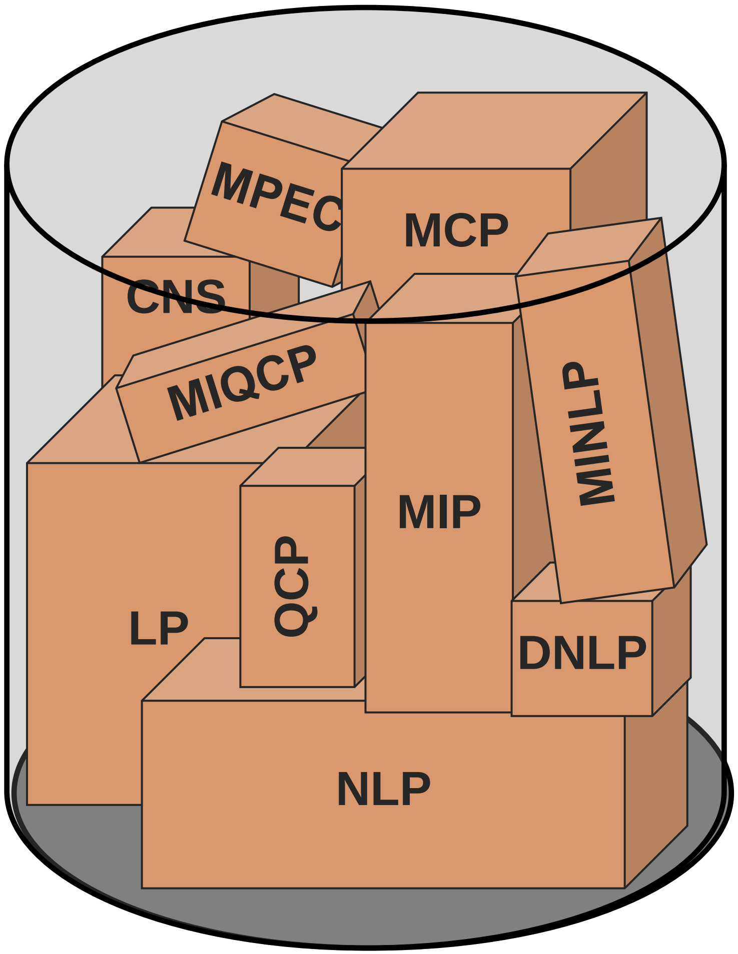
in fact only restricted by available memory

MIP

- Some (academic) problems still unsolvable
- Commercial problems mostly docile

NLP/MINLP

- Predictions are problem and data specific, global vs. local solutions



Non-traditional solution concepts

- MP with Equilibrium Constraints (MPEC)
  - ➢ **NLPEC**
    - Solves MPECs through reformulation into NLPs

- Solving non-integer models as MCPs
  - ➢ **PATHNLP**
    - reformulation via KKT conditions (1st and 2nd order deriv.)

- Mathematical Programming System for General Equilibrium analysis
  - ➢ **MPSGE**

- Indicator Constraints (**CPLEX**)
  – Alternative to conventional BigM formulations

Non-traditional solution concepts

- Global Optimization
  - ➢ **BARON, LINDOGLOBAL**
    - Proven global optimum
  - ➢ **LGO, OQNLP**
    - Stochastic convergence to global optimum

- Stochastic Programming
  - ➢ **DECIS**
  - – solves two-stage stochastic linear programs with recourse
  - – two-stage decomposition (Benders)
  - – stores only one instance of the problem and generates scenario sub-problems as needed
  - – solution Strategies (Universe problem/Importance sampling)

- …

New solution concepts

- Embedded Complementarity Systems
- Disjunctive Programs
- Bilevel Programs
- Extended Nonlinear Programs
- Variational Inequalities
- …

  - ➢ Breakouts of traditional MP classes
  - ➢ No conventional syntax
  - ➢ Limited support with common model representation
  - ➢ Incomplete/experimental solution approaches
  - ➢ Lack of reliable/any software

What now?

Do not:

- overload existing GAMS notation right away !
- attempt to build new solvers right away !

But:

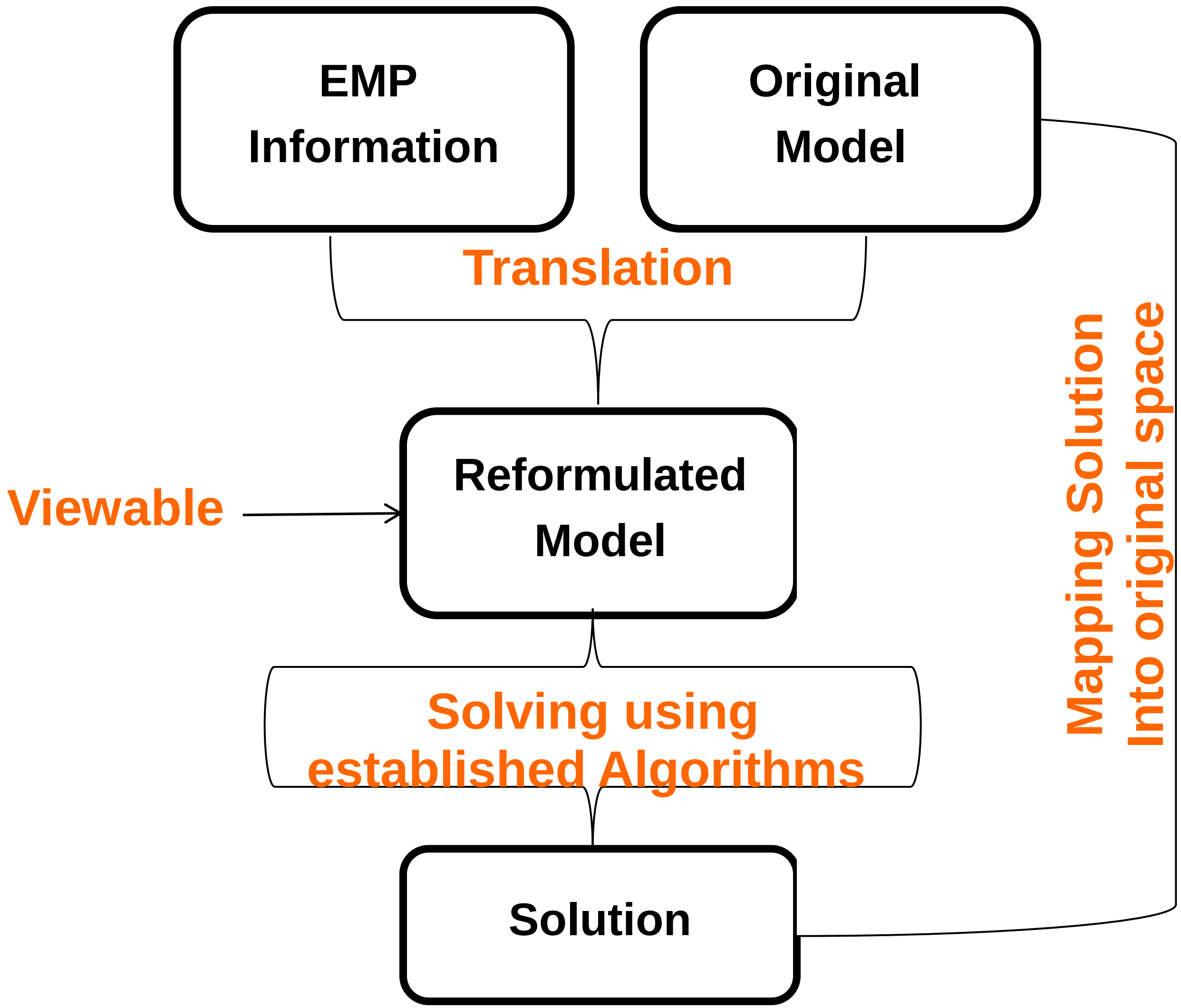
- Use existing language features to specify additional model features
- Distribute information as part of the production system
- Express extended model in symbolic form and apply existing matured solution technology

→ **Extended Mathematical Programming (EMP)**

Agenda

- **G**eneral **A**lgebraic **M**odeling **S**ystem
- New Solution Concepts
- **E**xtended **M**athematical **P**rogramming

GAMS “Solver” EMP



Embedded Complementarity Systems

$$
\begin{array}{llr}
\text{Max} & u_{11}x_{11} + u_{12}x_{12} + u_{21}x_{21} + u_{22}x_{22} & \text{Dual var.} \\
\text{s.t.} & x_{11} \qquad\qquad\qquad\qquad x_{21} \le \omega_{11} + \omega_{12} & p_1 \\
 & \qquad\qquad x_{12} \qquad\qquad\quad x_{22} \le \omega_{21} + \omega_{22} & p_2 \\
 & p_1 x_{11} + p_2 x_{12} \qquad\qquad\quad \le p_1\omega_{11} + p_2\omega_{12} & \lambda
\end{array}
$$

This is not an optimization model! How to solve?

GAMS

Embedded Complementarity Systems

- Write model as regular NLP with side constraints/variables

$$
\begin{aligned}
\min_x \quad & f(x,y) \\
\text{s.t.} \quad & g(x,y) \le 0 \quad (\perp \lambda \ge 0) \\
& \\
H(x,y,\lambda) = 0 & \quad (\perp y \text{ free})
\end{aligned}
$$

- Provide the additional information

```
$onecho  > %emp.info%
dualequ H y
dualvar λ g
$offecho
```

- EMP automatically creates the equivalent MCP model

$$
\begin{aligned}
\nabla_x \mathcal{L}(x,y,\lambda) & \quad \perp x \text{ free} \\
-\nabla_\lambda \mathcal{L}(x,y,\lambda) & \quad \perp \lambda \ge 0 \\
H(x,y,\lambda) = 0 & \quad \perp y \text{ free}
\end{aligned}
$$

ECS Example

```
parameter
        kterm              Terminal capital stock

UTIL..                 UTILITY =E= SUM(t, 10 * dfactor(t) * L(t) * LOG(C(t)/L(t)));
CC(t)..                   C(t) =E= Y(t) - I(t);
YY(t)..                   Y(t) =E= phi * L(t)**(1-kvs) * K(t)**kvs;
KK(t)..                   K(t) =L= (1-delta)**10 * K(t-1) + 10 * I(t-1) + kinit$tfirst(t);
TERMCAP..              kterm =E= sum(tlast, (1-delta)**10 * K(tlast) + 10 * I(tlast));

model ramsey NLP Model using parameter kterm /all/;

set iter /iter1*iter20/;

kterm = kinit * power(1+g,card(t));

parameter         invest(t,iter)   Investment in successive iterations
                  kt(iter)         Terminal capital stock in successive iterations;

loop(iter,
        kt(iter) = kterm;
        solve ramsey maximizing UTILITY using NLP;
        invest(t,iter) = I.L(t);

        kterm = sum(tlast(t), K.L(tlast) * Y.L(t)/Y.L(t-1));
);
```

(Thomas F. Rutherford)

EMP Formulation

```
*Substitute TERMCAP of NLP by TERMCAPV (using variable KTERMV instead of parameter kterm)
TERMCAPV..  KTERMV =E= sum(tlast, (1-delta)**10 * K(tlast) + 10 * I(tlast));

*First-order-condition for terminal capital stock variable
SSTERM.. sum(tlast(t),I(t)/I(t-1) - Y(t)/Y(t-1)) =E= 0;

model ramseynlpd /UTIL,CC,YY,KK,TERMCAPV,SSTERM/;

$onecho > %emp.info%
dualequ SSTERM KTERMV
$offecho

option nlp=emp;

solve ramseynlpd maximizing UTILITY using nlp;
```

```
Extended Mathematical Programming (EMP)
----------------------------------------
--- EMP Summary  (errors=0)
    Adjusted Equations = 0
    Dual Variable Maps = 0
    Dual Equation Maps = 1
    Bilevel Followers  = 0
    Disjunctions       = 0
--- The model C:\home\distrib\tvis_alpha\convtest\emp\225a\emp.scr will be solved by GAMS
---
```

Disjunction Example

A set of tasks is to be processed on a single machine.

- The execution of the tasks is non-preemptive (ie cannot be interrupted).

- Every task has a release date, duration and due date are given.

```
table data(times,job)
                1      2      3      4      5      6      7
release         2      5      4                    8      9
duration        5      6      8      4      2      4      2
due             10     21     15     10     5      15     22
```

- Objective: What is the sequence that minimizes the maximum tardiness?

Disjunction Example

```
seq(i,j)$(not sameas(i,j))..  comp(i) =l= start(j);
```

- Either has to hold for (i,j) or (j,i)
- How to model these disjunctions:
  - BigM Formulation
  - Convex Hull Formulation
  - Indicator constraints (CPLEX)
- Which is adequate/best formulation for my problem?

Hierarchical Models

- Bilevel Program:

$$\begin{aligned} \min_{x,y} \quad & f(x,y) \\ \text{s.t.} \quad & g(x,y) \leq 0, \\ & y \text{ solves } \min_{s} v(x,s) \text{ s.t. } h(x,s) \leq 0 \end{aligned}$$

- Additional Information:

```
$onecho  > %emp.info%
Bilevel x min v h
$offecho
```

- EMP Tool automatically creates an MPEC by expressing the lower level optimization problem through its optimality conditions

Bilevel Model

Conejo A J, Castillo E, Minguez R, and Garcia-Bertrand R; Decomposition Techniques in Mathematical Programming, Springer,Berlin, 2006.

```
variables z,x1,x2,x3,x4,h1,h2,u1,u2,u3,u4,v1,v2,v3,v4;
equations defobj,defh1,defh2,a1,e1,e2;

defobj.. z =e= sqr(x1+x2-2) + sqr(x3+x4-2);
a1.. x1-x2 =e= 3;
```

Outer Problem

```
defh1.. h1 =e= sqr(u1-x1) + sqr(u2-x2) + sqr(u3-x3) + sqr(u4-x4);
e1.. 3*u1 +  u2 + 2*u3 + u4 =e= 6;
```

Inner Problem 1

```
defh2.. h2 =e= sqr(v1-x1) + sqr(v2-x2) + sqr(v3-x3) + sqr(v4-x4);
e2.. v1 +  v2+ v3 + 2*v4 =e= 7;
```

Inner Problem 2

```
model bilevel / all /
```

EMP Information File + EMP Summary Log

```
option nlp=emp;

$onecho > %emp.info%
bilevel x1 x2 x3 x4
min h1 defh1 e1
min h2 defh2 e2
$offecho

solve bilevel us nlp min z;
```

```
Extended Mathematical Programming (EMP)
----------------------------------------
--- EMP Summary  (errors=0)
    Adjusted Equations = 0
    Dual Variable Maps = 0
    Dual Equation Maps = 0
    Bilevel Followers  = 2
    Disjunctions       = 0
--- The model C:\home\distrib\tvis_alpha\convtest\emp\225a\emp.scr will be solved by GAMS
---
```

Extended Nonlinear Programming

Soft penalization of constraints

- Model:

$$\begin{array}{ll} \min\limits_{x_1,x_2,x_3} & \exp(x_1) \\ \text{s.t.} & \log(x_1) = 1 \\ & x_2^2 \leq 2 \\ & x_1/x_2 = \log(x_3), 3x_1 + x_2 \leq 5, x_1 \geq 0, x_2 \geq 0 \end{array}$$

- Additional information:

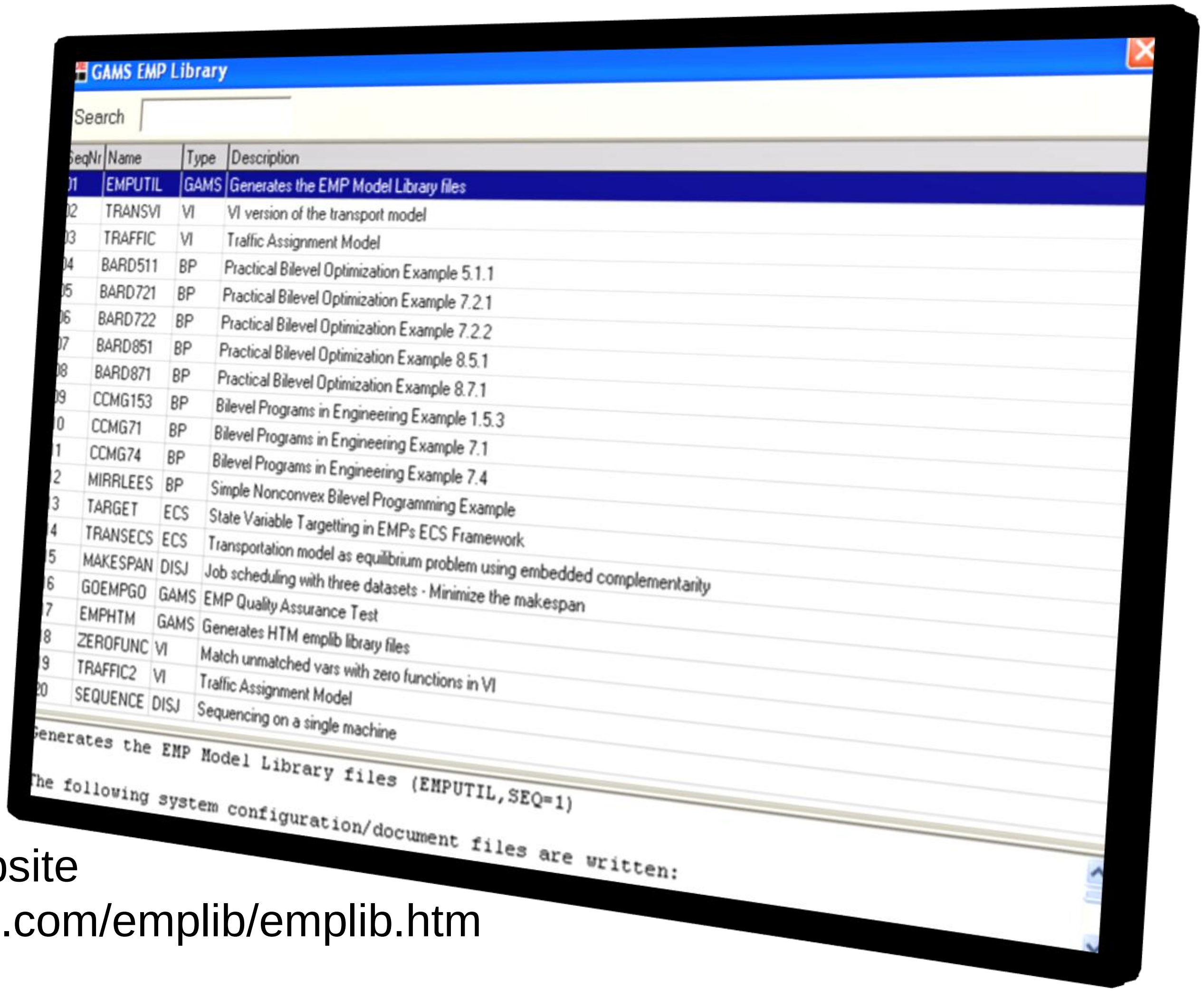
```
$onecho > %emp.info%
Adjustequ
e1 sqr   5
e2 MaxZ  2
$offecho
```

```
$onecho > %gams.scrdir%empinfo2.scr
Strategy MCP
Adjustequ
e1 sqr   5
e2 MaxZ  2
$offecho
```

- EMP Tool creates the NLP model (or the MCP via KKT) :

$$\begin{array}{ll} \min\limits_{x_1,x_2,x_3} & \exp(x_1) + 5\,\|\log(x_1) - 1\|^2 + 2\max(x_2^2 - 2, 0) \\ \text{s.t.} & x_1/x_2 = \log(x_3), 3x_1 + x_2 \leq 5, x_1 \geq 0, x_2 \geq 0 \end{array}$$

EMP Library



- Distributed with GAMS
- Available on website http://www.gams.com/emplib/emplib.htm

Conclusion

EMP

- automates symbolic reformulations to avoid error-prone and time-consuming manual algebra (re)writing
- offers solutions where solutions couldn't be offered before
- facilitates to compare concurrent strategies
- free
- But: non-exhaustive

Thank you !

Europe

GAMS Software GmbH
Eupener Str. 135-137
50933 Cologne
Germany

Phone: +49 221 949 9170
Fax: +49 221 949 9171
http://www.gams.de

info@gams.de
support@gams-software.com

USA

GAMS Development Corp.
1217 Potomac Street, NW
Washington, DC 20007
USA

Phone: +1 202 342 0180
Fax: +1 202 342 0181
http://www.gams.com

sales@gams.com
support@gams.com