



Stochastic Optimization: Recent Enhancements in Algebraic Modeling Systems

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GAMS at a Glance

The screenshot displays the GAMS software interface with the following components:

- Code Editor:** Contains GAMS code for creating an example GDX file for charting. Key lines include:


```

      * Create gdx file for charting demo
      * The generated gdx file can be used to fo
      *
      * GAMS Development Corporation, Formulation
      $stitle data for single lines, bars, pie
      set years y1998:y2005 /1
      parameter YearDataA(years), YearDataB(year:
      YearD:
      YearD:
      YearD:
      chartdat.gdx
      $stitle
      set
      param
      scalar
      delta
      loop(
      po:
      po:
      x:
      ):
      $stitle
      set
      set
      param
      vector
      
```
- Data Table:** A table listing model entries with columns for Entry, Symbol, Type, Dim, and Nr Elem.

Entry	Symbol	Type	Dim	Nr Elem
10	GanttData	Par	3	14
4	Points	Par	2	200
8	Scatter2D	Par	2	40
9	Scatter3D	Par	2	60
13	ScenarioData	Par	2	136,000
12	StockData	Par	3	800
11	Surface	Par	2	2,500
5	Vector2D	Par	2	80
6	Vector2Db	Par	2	80
7	Vector3D	Par	2	120
1	YearDataA	Par	1	8
2	YearDataB	Par	1	8
3	YearDataC	Par	1	8
- StockData Chart:** A line chart showing stock prices for IBM (red), DELL (green), HP (yellow), and SUN (blue) from 1998 to 2005. The y-axis ranges from 102 to 104.
- Surface Chart:** A 3D surface plot showing a sharp peak. The x-axis is labeled with s2, s5, s8, s12, s16, s20, s24, s28, s32, s36, s40, s45, s49. The y-axis ranges from -0.2 to 0.6.
- Log Window:** Shows the execution log for 'chartdat.gms', including start and stop times and file paths.

General Algebraic Modeling System:
 Algebraic Modeling Language,
 Integrated Solver, Model
 Libraries, Connectivity- &
 Productivity Tools

Design Principles:

- Balanced mix of declarative and procedural elements
- Open architecture and interfaces to other systems
- Different layers with separation of:
 - model and data
 - model and solution methods
 - model and operating system
 - model and interface



AML and Stochastic Programming (SP)

- Algebraic Modeling Languages/Systems good way to represent optimization problems
 - Algebra is a universal language
 - Hassle free use of optimization solvers
 - Simple connection to data sources (DB, Spreadsheets, ...) and analytic engines (GIS, Charting, ...)
- Large number of (deterministic) models in production
 - Opportunity for *seamless* introduction of new technology like Global Optimization, Stochastic Programming, ...
 - AML potential framework for SP



Stochastic Programming Claims and '*Facts*'

- Lots of application areas (Finance, Energy, Telecommunication)
- Mature field (Dantzig '55)
- Variety of SP problem classes with specialized solution algorithms (e.g. Bender's Decomposition)
- Compared to deterministic mathematical programming (MP) small fraction
 - Only 0.1% of NEOS submission to SP solvers
- No/few commercially supported solvers for SP
- Various frustrations with industrial SP projects



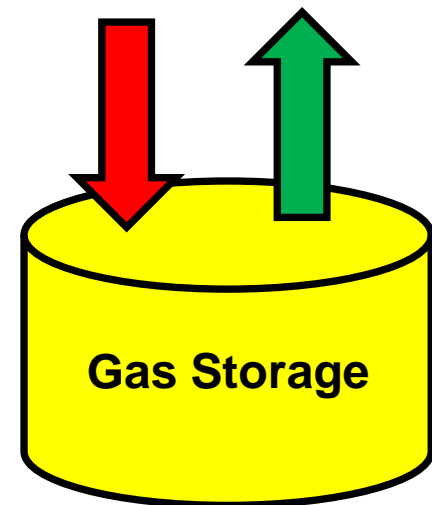
Example Model: Gas Price Model

Natural Gas NYMEX Weekly Price Chart



Inject/
Buy

Withdraw/
Sell



➔ [gas1.gms](#)
(deterministic model)



n-Stage Stochastic Programs

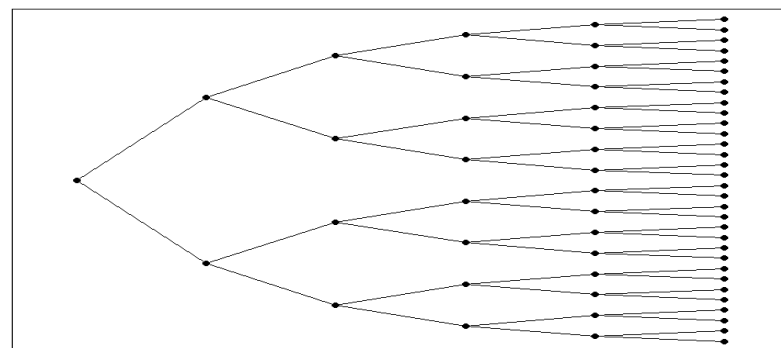
- Construct Scenario Tree:
 - Start with today's price and use a (discrete) distribution
 - Realizations: up, down
- Stochastic Linear Program (block structure)
 - Decomposition (Benders, SDP, SDDP, ...)
 - In practice Deterministic Equivalent with Barrier method

$$Z_{HN} = \min_{x_1} \left\{ c_1 x_1 + E_{\xi_2} \left[\min_{x_2} c_2 x_2 + E_{\xi_3 | \xi_2} \left[\min_{x_3} c_3 x_3 + \dots + E_{\xi_T | \xi_{T-1}, \dots, \xi_2} \min_{x_T} c_T x_T \right] \right] \right\}$$

subject to:

$$\begin{array}{rcccc} A_{11}x_1 & & & & = b_1 \\ A_{21}x_1 + & A_{22}x_2 & & & = b_2 \\ A_{31}x_1 + & A_{32}x_2 + & A_{33}x_3 & & = b_3 \\ \vdots & & & \ddots & \vdots \\ A_{T1}x_1 + & A_{T2}x_2 + & A_{T3}x_3 + & \dots & + A_{TT}x_T = b_T \end{array}$$

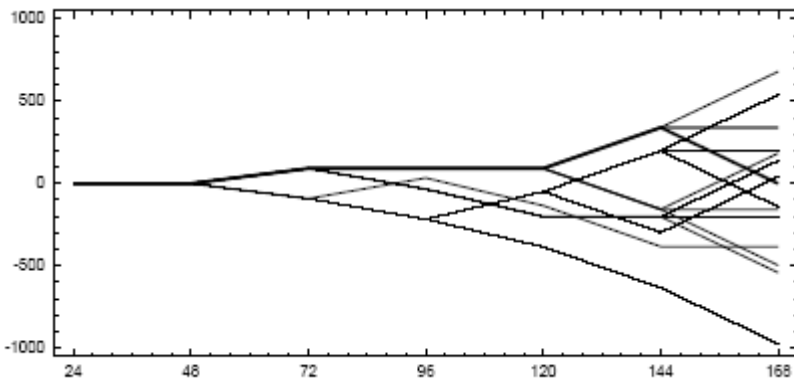
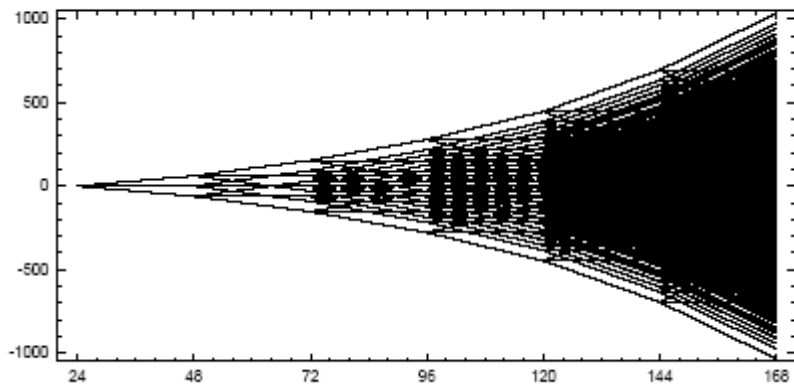
$$\ell_t \leq x_t \leq u_t;$$



⇒ [gas2.gms](#)
(n-stage SP, distribution)



ScenRed (Römisch et. al., HU Berlin)



- Find good approximation of original scenario tree of significant smaller size.
- Available since 2002
- Integrated in GAMS system
- No extra cost

[gas2.gms](#)

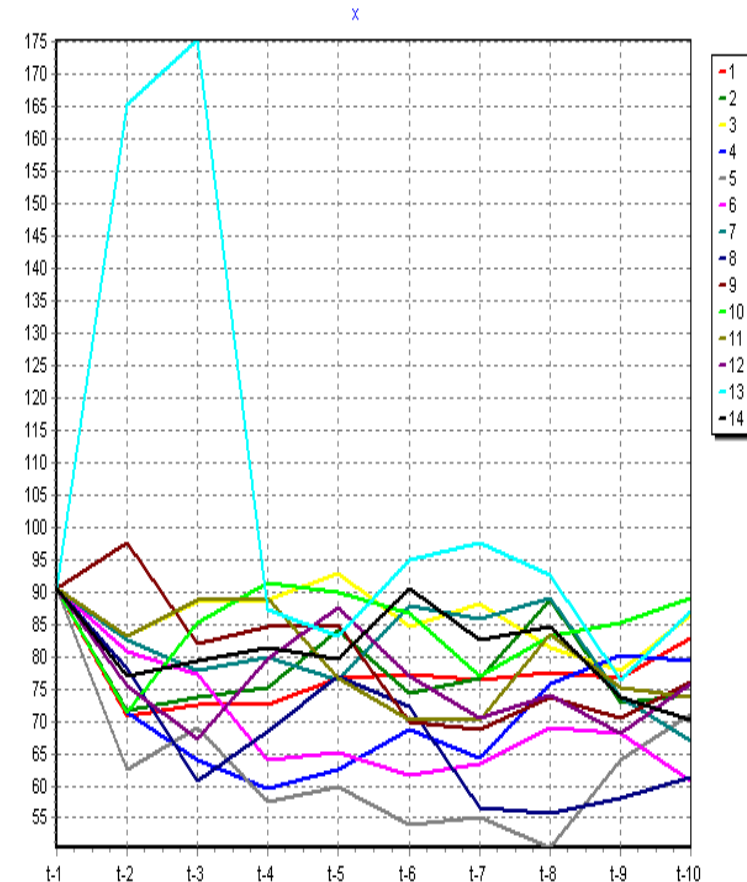
⇒ (n-stage SP, distribution)
plus ScenRed



Scenario based Stochastic Programs

- Random variables with distributions versus independent scenarios
- Wait-and-see (WS)
 - solve scenarios independently (grid computing)
- Expected value problem (EV)
 - Calculate EV of random variables and solve
- Expectation of EV problem (EEV)
 - Fix decisions of EV problem and evaluate

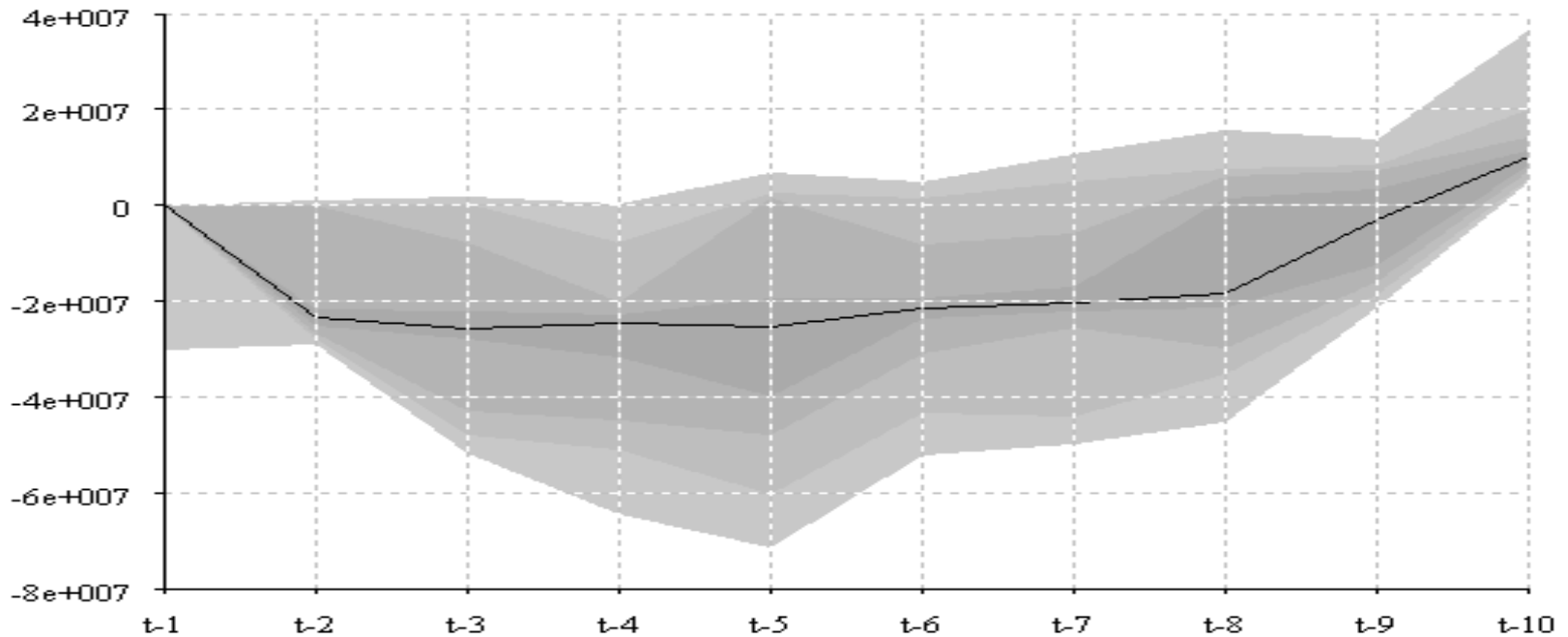
⇒ [gas3.gms](#) (Scenario based: WS, EEV)





Value of Stochastic Solution/Visualization

- $WS \geq EEV$ (maximization!)
- Visualize results!
 - e.g. fan plot (Tom Rutherford, ETH Zürich)

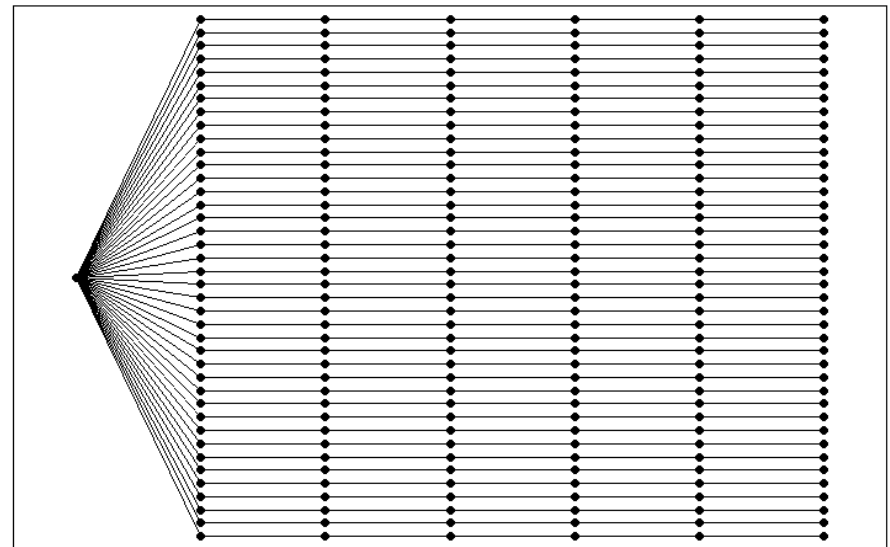


⇒ [gas3.gms](#) (Scenario based: fan plot)



2-Stage Stochastic Programs

- SP Solver DECIS (Gerd Infanger, Stanford, USA)
 - Stores only one instance of the problem and generates scenario sub-problems as needed
 - Solution Strategies
 - Universe problem (all scenarios)
 - Sampling: Crude Monte Carlo/Importance sampling

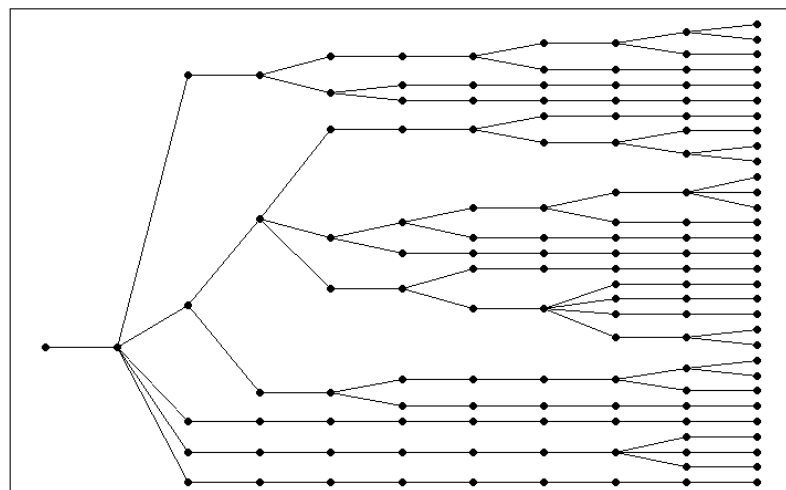
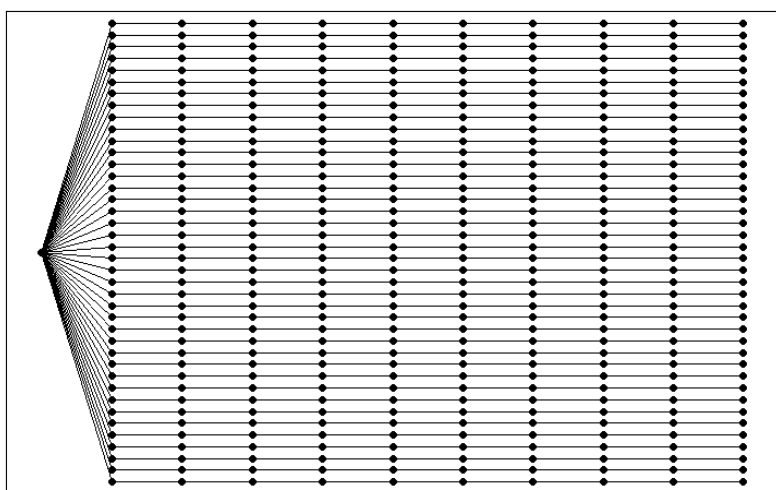


⇒ [gas4.gms](#) (2-stage, DECIS)



Tree Generation: ScenRed2

- Construct a true scenario tree from independent scenarios:



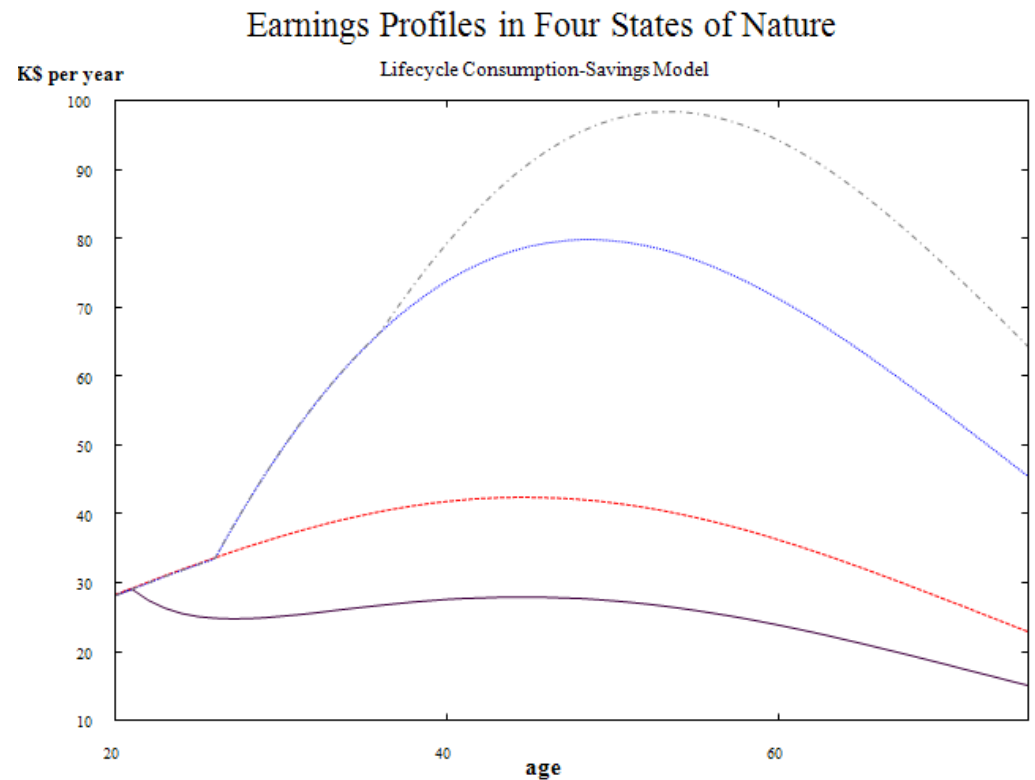
- Reconstruct underlying distribution from a set of scenarios
- Here-and-now (HN): $WS \geq HN \geq EEV$
- Value of stochastic solution $VSS = HN - EEV$
- Expected value of perfect information $EVPI = WS - HN$

⇒ [gas5.gms](#) (n-stage, ScenRed2)



The Rich World of Stochastic Programming

- n-stage stochastic linear programming (SLP) just one option
- SP models from application areas exist (finance)
- Economic modeling
 - mixed complementarity problems
 - scenario trees with few branches





Conclusion

- Stochastic Programming still challenging and developing field
 - GUPOR: *Uncertainty: An OR Frontier* (Greenberg, 2006)
- Lack of solution technology limits the dissemination of SP
- There is more to SP than n-stage SLP
- Representation of results
- Collection of comprehensive & reproducible examples could help to *spread the word*



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