Modeling with Quadratic Constraints to Improve Exam Timetabling Solutions

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Agenda

- Overall Idea of our approach
- Research incentive: The Exam Timetabling Project
- Literature in Exam Timetabling
- The Original MILP model
- The new MINLP model with Quadratic Constraints
- Solution Improvement Approach
- Computational Results
- Summary and Conclusion

Overall Idea: Variable Redefinition and Reformulation in Quadratic Constraints

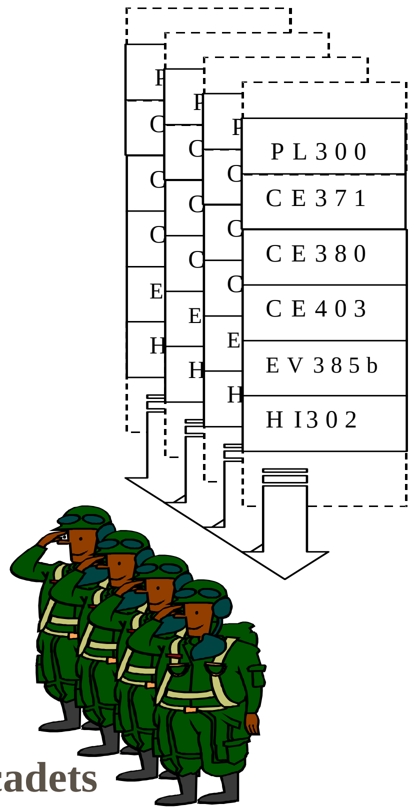
Different with the traditional linearization idea in integer program!

Research incentive: The Exam Timetabling Project and its features

Exam Timetabling Problem at USMA –West Point

	1	2	…	6
Morning session	CE371 CH384 CS383 HI366	CH101 CS408 EE301 EN302		EV203 PH203 PL300 LR204
afternoon session	CE404 LG484 LS362 MS350	LF382 SE388 SS388 …		CE403 CS380 SS201 …

Term End Exam Schedule (≈250 exams)



≈4000 cadets
≈18000 enrollments

Basic Requirements

- Assignment
- No resource conflict (Clash free)
- Specifying part of the solution (Fixing)
- Excluding part of the solution space (Prohibited periods)

Special USMA requirements

- Consecutive (back to back) exam limit
- Inclusive/Exclusive groups (Courses that have to be scheduled together/apart)
- Plebe (Firsties) constraint
  - One exam a day for Plebes
  - Exam period is over by period X for Firsties

Infeasibility And Makeup For Exams

Difficulty at USMA Term End Exam Scheduling

- **The exam timetabling problem at USMA is a NP-complete problem**
- **Infeasibility with some of the requirements:**
  consecutive, exclusive, fixed…
- **High conflict density**
- **Small number of available periods**

Literature in Exam Timetabling

Techniques

- Clustering
- Sequencing
- Graph coloring
- Local search (GA, Tabu, simulated annealing,
- CSP and logic programming

Research field related

- Operations research/mathematical programming
- Artificial intelligence (AI)
- Human-machine interaction (some scheduling strategies have relied on interactive human aid)

The Original MILP model

(W) **Min z** $= \sum_{r \in R, p \in P} y(r,p)$ **- |R|** **minimizing makeup**

subject to

$$\sum_{p \in P} x(c,r,p) = 1 \qquad \forall (c,r) \in CR \subseteq (C \times R)$$ **(1)- cadet assignment**

$$\sum_{r \in R(c)} x(c,r,p) \le 1 \qquad \forall c \in C, p \in P$$ **(2)- no-conflict**

+ other requirement constraints

(3)- course opening
(4) – primary enrollment
(5) – one primary
(6)- exclusive (r_1,r_2)
(7)- inclusive (r_1,r_2)
(8)- consecutive constraint
(9)- no makeup constraint
(10)- Plebe
(11) - fixed
(12) -prohibit
(13) – completion

The New Quadratic Constraint Model – Variable Redefinition

Original model:

$X(c,r,p)$ = 1 if cadet $c \in C$ is scheduled in exam $r \in R$ at period $p \in P$

$Y(r,p)$ = 1 if exam $r \in R$ is scheduled in period $p \in P$

New model

$\bar{x}(c,r,m)$ =1 if cadet $c \in C$ is scheduled to take exam $r \in R$ in session $m \in M$

$\bar{y}(r,m,p)$ =1 if session $m \in M$ of exam $r \in R$ is scheduled in period $p \in P$.

Relationship:

$$x(c,r,p) = {}_{m}\bar{x}(c,r,m)\bar{y}(r,m,p)$$

and $$y(r,p) = \max_{m \in M} \bar{y}(r,m,p)$$

Old vs. New Model

(w) Min z $= \sum_{r \in R, p \in P} y(r,p)$ - |R|

s.t. $\sum_{p \in P} x(c,r,p) = 1 \quad \forall (c,r) \in CR \subseteq (C \times R)$ (1)- cadet assignment

$\sum_{r \in R(c)} x(c,r,p) \leq 1 \quad \forall c \in C, p \in P$ (2)- no-conflict

+ other requirement constraints ……

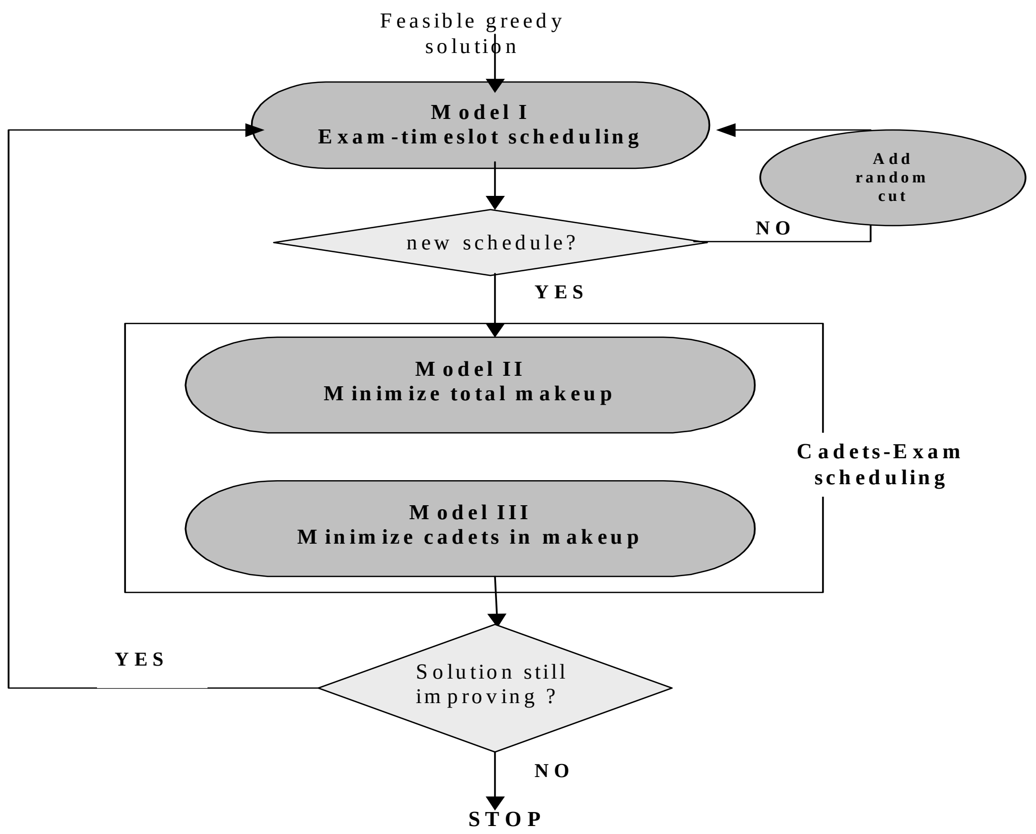
$(\overline{w})$ Min z $= \sum_{r \in R, p \in P} y(r,p)$ - |R|

s.t. $\sum_{p \in P} \sum_{m \in M} \overline{x}(c,r,m)\overline{y}(r,m,p) = 1 \quad \forall (c,r) \in CR \subseteq (C \times R)$ (1)- cadet assignment

$\sum_{r \in R(c)} \sum_{m \in M} \overline{x}(c,r,m)\overline{y}(r,m,p) \leq 1 \quad \forall c \in C, p \in P$ (2)- no-conflict

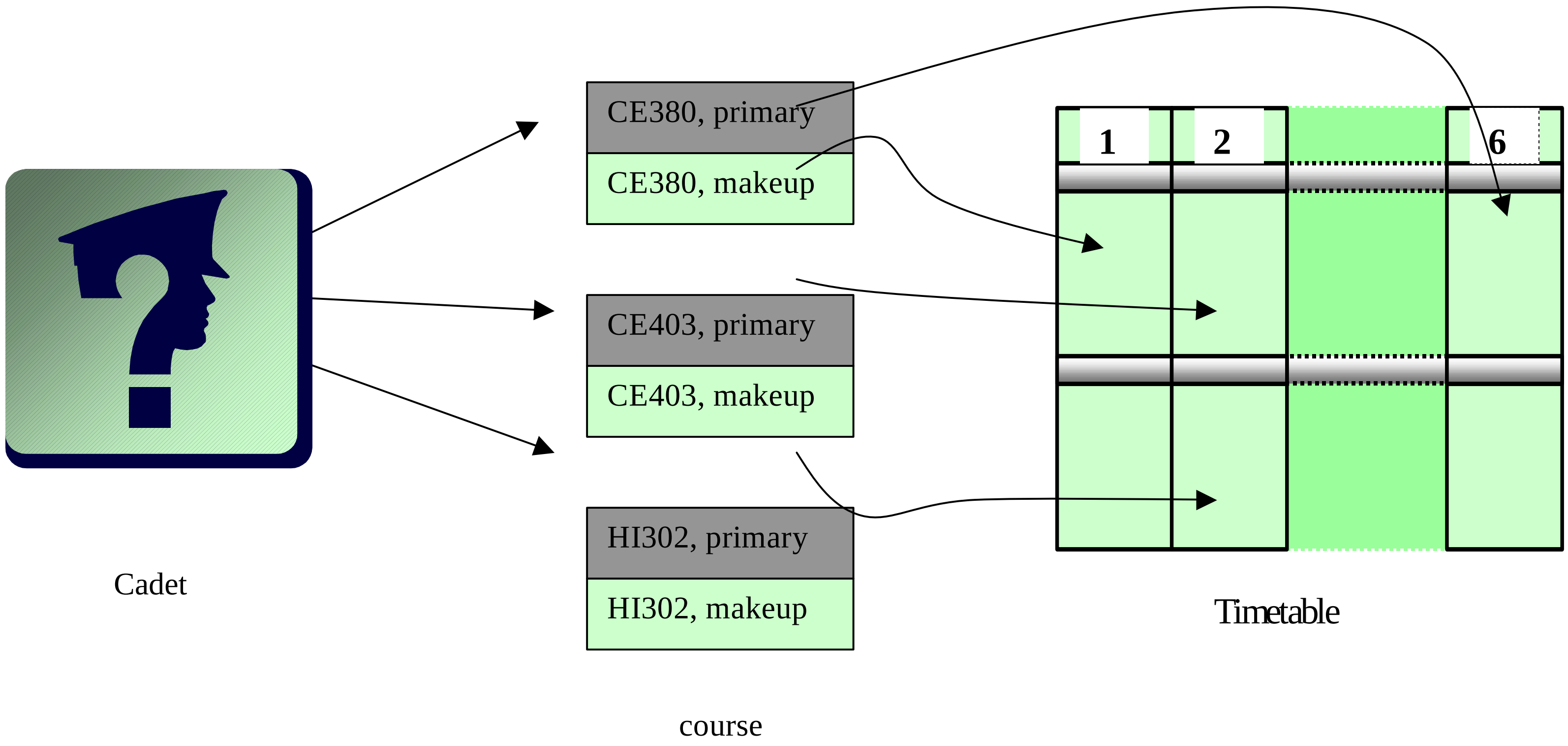
+ other requirement constraints ……

The Decomposition Approach And Iterative Algorithm



Intuition of the Solution Improvement Approach

- **Given a feasible solution**
  - Assignment of exam courses (primary and makeup) to periods
  - Assignment of cadets to periods
- **Decompose the problem:**



Computational Result(1)

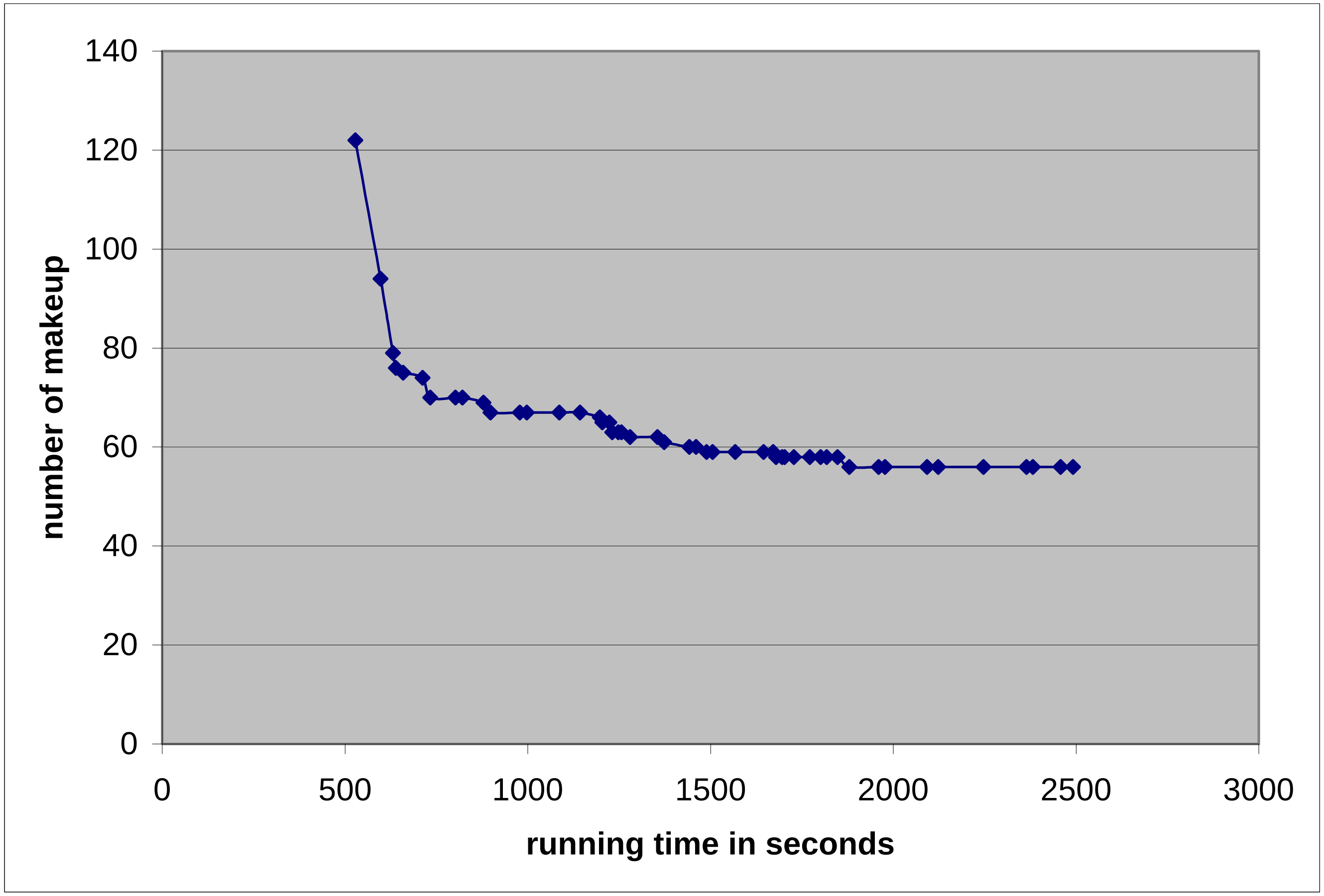
TEE Schedule for AY2001/1

- **Chuck + Legacy system**
  - Partial schedule, approx. 90 makeups (4 Weeks)
- **Chuck + GAMS TEE scheduler**
  - Complete schedule, no conflicts, 60 makeups (10 minutes)
  - The improver module produced schedule with 40 makeups

Computational Result(2)

	initial solution		improved solution		
instance	time	makeups	time	makeups	%
1_0	634	104	632	63	39.42
1_1	508	122	622	67	45.08
1_2	485	130	620	61	53.08
1_3	651	131	619	58	55.73
1_4	556	128	626	61	52.34
1_5	521	124	605	65	47.58
2_0	223	60	472	49	18.33
2_1	190	69	303	31	55.07
2_2	165	61	399	45	26.23
2_3	122	65	218	47	27.69
2_4	119	65	360	48	26.15
2_5	179	57	302	46	19.30
3_0	276	57	327	38	33.33
3_1	359	78	621	36	53.85
3_2	238	55	418	34	38.18
3_3	238	55	606	33	40.00
3_4	198	49	478	32	34.69
3_5	170	68	420	31	54.41

Computational Result(2)



Summary and Conclusion

We expect that this approach can be used for other difficult scheduling tasks, where a hierarchy of decisions may lead to similar Quadratic/bilinear remodeling and separation of the problem into individually and sequentially solvable subproblems

The End